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PRINCETON UNIVERSITY

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TANDEM HELICOPTER LATERAL STABILITY AND CONTROL

Aeronautical Engineering Laboratory
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Summary

The equations of motion, for lateral-directional dynamic stability and response of tandem helicopters in forward flight, are formulated. Theoretical expressions are developed for the various stability derivatives, and the results of preliminary calculations for an HUP type aircraft are discussed.

Introduction

This report is the result of preliminary studies of lateral-directional dynamic stability characteristics of tandem helicopters in forward flight. The work is part of the third phase of a program of theory and static and dynamic flight testing to develop a practical working method of stability estimation for tandem helicopters. The first two phases, involving longitudinal stability are essentially complete (Ref. 1 and 2). The work reported here is to be followed by analysis of static lateral flight tests to verify the theoretical lateral derivatives, and then by lateral dynamic response tests for correlation of those characteristics with theoretical predictions.

Axis Systems and Notation

The equations of dynamic motion developed herein are in terms of forces and moments along and about body axes fixed in the aircraft, and rotating and translating with it. Their origin is at the aircraft C.G.; the X axis lies in the plane of symmetry, perpendicular to the rotor shafts; and the Z axis is parallel to the drive shafts.

It is convenient to use existing expressions for the rotor forces and moments in terms of axes set with the local wind seen by the rotors. Quantities measured in this "wind" axis system are subscripted $(\ )_w$: such as T_w, acting along the axis of No-Feathering (ANF); and the side drag forces, Y_w and H_w, acting in directions perpendicular and parallel to the plane containing ANF and the instantaneous relative wind.

In relating the two above systems, it is convenient to use an intermediate system subscripted $(\ )_i$, in which T_i is along the ANF, and Y_i and H_i are perpendicular and parallel to the plane of the ANF and the X body axis.

In the body axis system, of course, T acts along the rotor drive shafts, and Y and H are perpendicular and parallel to the X axis.

Symbols used in the theoretical analysis are detailed below:

General

$(\ )'$	Primed quantities refer to the rear rotor.
$\pm(\ )$	The upper of the two signs preceding a quantity refers to the front and the lower to the rear rotor value.
$\Sigma(\ )$	Algebraic sum of the front and rear rotor contributions of a given quantity.
$\Delta(\ )$	Difference between fromt and rear rotor contributions of a given quantity, the rear rotor contribution being subtracted from the front.
λ	Root of characteristic equation.
$C_{1,2,3\cdots}$	Coefficients of characteristic equation.
R	Routh's Discriminant for characteristic equation.
$(\bar{\ })$	Bar denotes $\frac{\partial}{\partial\alpha}(\ )$.
$(\ )_f = (\ )_{fus}$	Subnotation denotes fuselage.
$(\ )_x$	Subscript, following standard practice, denotes partial differentiation with respect to subscript quantity; $(\ )_x = \frac{\partial(\ )}{\partial x}$.
$(\dot{\ })$	Dot denotes differentiation with respect to time; $(\dot{\ }) = \frac{d}{dt}(\ )$.
$D(\ )$	Derivative with respect to non-dimensional time; $D(\ ) = \frac{d(\ )}{d(\frac{t}{\tau})}$

Forces and Moments

T Rotor thrust, or, upward component of rotor resultant force (lbs.).

H Horizontal component of rotor resultant force, directed aft (lbs.).

Y Side component of rotor resultant force, positive to the right (lbs.).

L, N Rolling and yawing moments of helicopter about C.G.; roll is positive for right side down and yaw for nose right (ft.-lbs.).

C_T Thrust coefficient. $C_T = T/\rho\pi R^2(\Omega R)^2$

C_H Horizontal force coefficient. $C_H = H/\rho\pi R^2(\Omega R)^2$

C_Y Side force coefficient. $C_Y = Y/\rho\pi R^2(\Omega R)^2$

C_ℓ Rolling moment coefficient. $C_\ell = L/\rho\pi R^2(\Omega R)^2\ell$

C_n Yawing moment coefficient. $C_n = N/\rho\pi R^2(\Omega R)^2\ell$

$(C_Y)_f$ Fuselage side force coefficient. $(C_Y)_f = Y_f/\rho\pi R^2 V^2$

$(C_\ell)_f$ Fuselage rolling moment coefficient. $(C_\ell)_f = L_f/\rho\pi R^2 V^2\ell$

$(C_n)_f$ Fuselage yawing moment coefficient. $(C_n)_f = N_f/\rho\pi R^2 V^2\ell$

Physical Characteristics

b Number of blades per rotor.

c Average blade chord (ft.).

e Flapping hinge offset, distance from blade flapping hinge axis to rotor shaft centerline (ft.).

h_o Average height of front and rear rotors, measured from fuselage longitudinal axis (ft.).

I_{Blade} Mass moment of inertia of blade about the flapping hinge (slug-ft.^{2})

$$I_{Blade} = \int_e^R r^2\, dm$$

I_x, I_y, I_z Mass moments of inertia of helicopter about its x, y, and z body axes respectively.

I_{xz} Mass product of inertia of helicopter for the x and z body axes.

$k_{(\ )}$ Radius of gyration of helicopter about its ( ) body axis. $k_{(\ )}^2 = \frac{I_{(\ )}}{m}$

K_a Linkage term representing radians of swashplate tilt per inch of lateral stick motion.

K_r Linkage term representing radians of swashplate tilt per inch of rudder pedal displacement.

l Distance between rotor shafts (ft.).

l_o Average distance from C.G. to the rotor shafts. $l_0 = \frac{1}{2} l$ (ft.)

m Mass of helicopter (slugs).

$M_{s\,blade}$ Static mass moment of blade about the flapping hinge (slug-ft.)

$$M_s = \int_e^R r\,dm$$

r Radius of blade station or element.

R Blade radius (ft.).

S_t Area of vertical tail surface.

W Weight of helicopter (lbs.).

<u>Velocities and Angles</u>

V Relative wind velocity (ft./sec.).

Ω Velocity of rotation of rotor, clockwise rotation for front rotor and counterclockwise for rear. (rad./sec.)

α Angle of attack; defined for body axis system as the angle between the relative wind and a normal to the rotor shaft, and for wind axis system, as between the relative wind and a normal to the ANF.

$\beta(\ )$ Sideslip angle of helicopter, positive for slip to right (rad.).

ϕ, ψ Roll, positive for right side down, and yaw, positive for nose to right, angles of helicopter, respectively (rad.).

p, r Non-dimensionalized roll and yaw rates, respectively.

$$p = \frac{l_o \dot{\phi}}{\Omega R} \qquad r = \frac{l_o \dot{\psi}}{\Omega R}$$

a_1 Longitudinal tilt of tip path plane, positive aft for both front and rear rotors (rad.).

b_1 Lateral tilt of tip path plane, positive to left for front rotor and to right for rear (rad.).

β_o Coning angle (rad.).

A_{1s} Lateral cyclic pitch of front rotor, positive to right (rad.).

A'_{1s} Lateral cyclic pitch of rear rotor, positive to right (rad.).

B_{1s} Longitudinal cyclic pitch of front rotor, positive forward (rad.).

B'_{1s} Longitudinal cyclic pitch of rear rotor, positive forward (rad.).

~~θ_o Collective pitch angle of rotor blades (rad.).~~

$A_{1s} = \pm K_a \delta_a$ (See section on "Control Effectiveness", equations 13, etc.) Incremental lateral cyclic pitch caused by positive control stick displacement δ_a (rad.).

δ_a Lateral control stick displacement, positive to right (in.).

$A_{1s} \mp K_r \delta_r$ (See section on "Control Effectiveness", equations 14, etc.) Incremental lateral cyclic pitch caused by positive rudder displacement δ_r (rad.).

δ_r Rudder control displacement, positive left pedal forward (in.).

Aerodynamic and Non-dimensional Parameters

μ Advance ratio. $\mu = \frac{V}{\Omega R}$

λ Rotor inflow ratio.

ρ Air density (slugs/ft.^{3})

a Slope of blade two-dimensional lift curve.

a_t Lift curve slope of vertical tail surface.

δ Profile drag coefficient of rotor blades.

ϵ/ϵ_o Induced velocity factor to account for the effect of front and rear rotor flow interference.

K Aerodynamic factor to account for the change in lateral flapping due to a triangular fore and aft induced velocity distribution.

σ Rotor solidity. $\sigma = \frac{bc}{\pi R}$

γ Locke's blade inertia coefficient. $\gamma = \frac{\rho a c R^4}{I_{Blade}}$

γ_2 Blade mass constant. $\gamma_2 = \frac{\rho a c R^3}{M_s}$

τ Non-dimensional time parameter. $\tau = \frac{m}{\rho \pi R^2 \Omega R}$

μ_* Helicopter density parameter. $\mu_* = \frac{m}{\rho \pi R^2 l_o}$

h_x Non-dimensional roll inertia parameter. $h_x = \frac{1}{a\sigma\mu_*}\left(\frac{k_x}{l_o}\right)^2$

h_z Non-dimensional yaw inertia parameter. $h_z = \frac{1}{a\sigma\mu_*}\left(\frac{k_z}{l_o}\right)^2$

h_{xz} Non-dimensional product of inertia parameter. $h_{xz} = \frac{1}{a\sigma}\left(\frac{I_{xz}}{\mu_* m l_o^2}\right)$

Analysis

Rotor Forces and Axis Transformations

The rotor forces are calculated on the basis of several simplifying assumptions, as follows:

1. The induced velocity over each rotor disc is constant. The interference induced velocities are accounted for in an approximate manner, based on longitudinal stability studies. The importance of these assumptions is less than in the longitudinal case, and they are believed valid where only perturbations of side forces are involved.

2. Assumptions of quasi-static blade motion are adopted. The blade dynamic modes of motion are thereby ignored, except for the gyroscopic lag damping effects. The effects of these assumptions are quite generally small, since the modes ignored are of very high frequency compared to the aircraft's characteristic modes.

3. Small-angle assumptions are generally adopted, to be consistent with the small perturbation, linearized equations to be developed. Certain higher order μ terms are seen in the development to be small, and are ignored in the interests of simplicity.

The basic equations for the rotor forces, in the wind axis system, are given below (see Ref. 3). Where double signs are used, the upper ones apply for the front rotor (clockwise rotation), and the lower for the rear rotor (counter-clockwise rotation).

1a) $$\frac{2(C_T)_W}{a\sigma} = (\bar{C}_T)_W \doteq \frac{1}{3}(\theta_0)_W + \frac{1}{2}(\lambda)_W$$

1b) $$\mp \frac{2(C_Y)_W}{a\sigma} = \mp(\bar{C}_Y)_W \doteq \Big| -\frac{3}{4}\mu\theta_0\beta_0 + \frac{1}{3}\theta_0 b_1 + \frac{3}{8}\mu^2\theta_0 b_1 + \frac{3}{4}\lambda b_1 + \frac{1}{6}\beta_0 a_1 - \frac{3}{2}\mu\lambda\beta_0 - \mu^2\beta_0 a_1 + \frac{1}{4}\mu a_1 b_1 + \frac{1}{8}\mu^2\lambda b_1 \Big|_W$$

1c) $$\frac{2(C_H)_W}{a\sigma} = (\bar{C}_H)_W \doteq \Big| \frac{\delta}{2a}\mu + \frac{1}{3}\theta_0 a_1 - \frac{1}{2}\mu\lambda\theta_0 + \frac{3}{4}\lambda a_1 + \frac{1}{4}\mu a_1^2 - \frac{1}{6}\beta_0 b_1 + \frac{1}{4}\mu\beta_0^2 + \frac{1}{8}\mu^2\theta_0 a_1 + \frac{1}{8}\mu^2\lambda a_1 - \frac{\phi_2}{2}\left(\frac{\mu a_1}{8} + \frac{\theta_0}{3\Omega} + \frac{\lambda}{\Omega}\right) \Big|_W$$

where

1d) $$\lambda_W = \mu\alpha_W - \frac{(C_T)_W}{2\mu}\left(1 + \frac{\epsilon}{\epsilon_0}\right)$$

The given equations (1) reflect, of course, the induced velocity assumptions, and the ϵ/ϵ_o factor in (1d) allows for rotor interference. These factors must be obtained emperically (HUP interference characteristics are contained in Ref. 1) and they represent the increase in rear rotor downwash due to the front rotor and the smaller upwash component on the front due to the rear rotor.

Similarly, blade flapping is given by

2a) $$(\beta_o)_w \doteq \gamma\left(\frac{\theta_o}{8} + \frac{\lambda}{6}\right)_w$$

2b) $$(a_1)_w \doteq \left|\frac{\frac{2}{3}\mu\theta_o + \frac{1}{2}\mu\lambda}{\frac{1}{4} - \frac{1}{8}\mu^2}\right|_w$$

2c) $$(b_1)_w \doteq \left|\frac{\frac{1}{3}\mu\beta_o K}{\frac{1}{4} + \frac{1}{8}\mu^2}\right|_w \pm \frac{16}{\gamma\Omega}\dot{\phi}$$

Equations (2) of course, involve the quasi-static flapping assumptions, and the factor K in equation (2c) allows for a triangular fore-aft distribution of induced velocity and its effect on lateral flapping. The factor K is given by

$$K = 1 + \frac{3}{8}\frac{(C_T)_w}{\mu^2\beta_o}$$

Equations for the transformation of forces between the axis systems, appropriately linearized, are

3a) $$T_i = T_w$$

3b) $$Y_i \doteq Y_w - H_w\beta_i$$

3c) $$H_i \doteq H_w + Y_w\beta_i$$

and

4a) $$T \doteq T_i = T_w$$

4b) $$Y \doteq Y_i + T_i A_{1s} \doteq Y_w - H_w\beta_i + T_w A_{1s}$$

4c) $$H \doteq H_i - T_i B_{1s} \doteq H_w + Y_w\beta_i - T_w B_{1s}$$

Aerodynamic angle transformations are

5a) $$\alpha_W \doteq \alpha_s - \beta_i A_{1s} - B_{1s}$$

5b) $$\beta_i \doteq \beta_s \pm \frac{l_o}{V}\dot{\psi} + \frac{h_o}{V}\dot{\phi}$$

Blade flapping transformations are, similarly:

6a) $$a_{1s} \doteq (a_1)_i - B_{1s} \doteq (a_1)_w - B_{1s} \mp (b_1)_w \beta_i$$

6b) $$(a_1)_i \doteq (a_1)_w \mp (b_1)_w \beta_i$$

6c) $$b_{1s} = (b_1)_i \mp A_{1s} \doteq (b_1)_w \mp A_{1s} \pm (a_1)_w \beta_i$$

6d) $$(b_1)_i \doteq (b_1)_w \pm (a_1)_w \beta_i$$

The preceding equations are sufficient for the evaluation of the derivatives of the rotor forces with lateral flight variables. These derivatives are to be ~~taken at the initial values of the variables -- for example:~~

$$(\beta_s)_o = \dot{\phi}_o = \dot{\psi}_o = 0$$

Before proceeding with the derivatives, however, it will be convenient to express the inflow ratio, λ , in terms of the flight variables in the body axis system. Using equations (1a) and (1d):

7a) $$\lambda_w = \frac{\mu \alpha_w - \frac{a\sigma}{12\mu} \theta_o \left(1 + \frac{\epsilon}{\epsilon_o} \right)}{1 + \frac{a\sigma}{8\mu} \left(1 + \frac{\epsilon}{\epsilon_o} \right)}$$

Substituting (5a) and (5b) into (7a):

7b) $$\lambda_w = \frac{1}{1 + \frac{a\sigma}{8\mu} \left(1 + \frac{\epsilon}{\epsilon_o} \right)} \left\{ \mu \left[\alpha_s - A_{1s} \left(\beta_s \pm \frac{l_o}{V} \dot{\psi} + \frac{h_o}{V} \dot{\phi} \right) - B_{1s} \right] - \frac{a\sigma}{12\mu} \theta_o \left(1 + \frac{\epsilon}{\epsilon_o} \right) \right\}$$

Rotor Side Force Derivatives

For the side-force derivative with sideslip, from (4b):

8a) $$\frac{\partial \bar{C}_Y}{\partial \beta_s} = \frac{\partial (\bar{C}_Y)_w}{\partial \beta_s} - (\bar{C}_H)_w + A_{1s} \frac{\partial (\bar{C}_T)_w}{\partial \beta_s}$$

Since $(\bar{C}_Y)_w$ is a function of β_o, b_1, a_1, and λ, which in turn are affected by β_S, the first term of (8a) can be expressed as

8b) $$\frac{\partial(\bar{C}_Y)_w}{\partial\beta_S} = \frac{\partial\lambda}{\partial\beta_S}\left\{\frac{\partial\bar{C}_Y}{\partial\lambda} + \frac{\partial\bar{C}_Y}{\partial a_1}\frac{\partial a_1}{\partial\lambda} + \frac{\partial\beta_o}{\partial\lambda}\left[\frac{\partial\bar{C}_Y}{\partial\beta_o} + \frac{\partial\bar{C}_Y}{\partial b_1}\frac{\partial b_1}{\partial\beta_o}\right]\right\}_w$$

which, from (1b), (2a), (2b), (2c), and (7b) becomes

8c) $$\mp\frac{\partial(\bar{C}_Y)_w}{\partial\beta_S} \doteq \frac{-\mu A_{1S} F}{1+\frac{a\sigma}{8\mu}\left(1+\frac{\epsilon}{\epsilon_o}\right)}$$

where

8d) $$F = \mu\beta_o\left(K-\frac{7}{6}\right)+\frac{\gamma}{6}\left[\frac{4}{9}\mu\theta_o\left(K-\frac{27}{16}\right)+\frac{a_1}{6}+\mu\lambda\left(K-\frac{3}{2}\right)\right]$$

In the above (8c) and (8d), certain higher order terms have been dropped. Note that since derivatives are to be evaluated at the initial condition of symmetrical, steady flight, no distinction of axis system for a_1 or λ is necessary in these equations.

The second term of (8a) is to be evaluated by (1c), which, however, may be simplified to

8e) $$(\bar{C}_H)_W \doteq \frac{\delta}{2a}\mu + \frac{1}{3}\theta_0 a_1 - \frac{1}{2}\mu\lambda\theta_0 + \frac{3}{4}\lambda a_1 + \frac{1}{4}\mu a_1^2$$

The last term of (8a) is

8f) $$\frac{\partial(\bar{C}_T)_W}{\partial\beta_s} = \left|\frac{\partial\bar{C}_T}{\partial\lambda}\frac{\partial\lambda}{\partial\beta_s}\right|_W = \frac{-\frac{1}{2}\mu A_{1s}}{1+\frac{a\sigma}{8\mu}\left(1+\frac{\epsilon}{\epsilon_0}\right)}$$

Using (8c) and (8f) in (8a), and including both rotors,

8g) $$\frac{\partial\bar{C}_Y}{\partial\beta_s} = \mu\Delta\left[\frac{FA_{1s}}{1+\frac{a\sigma}{8\mu}\left(1+\frac{\epsilon}{\epsilon_0}\right)}\right] - \frac{1}{2}\mu\Sigma\left[\frac{A_{1s}^2}{1+\frac{a\sigma}{8\mu}\left(1+\frac{\epsilon}{\epsilon_0}\right)}\right] - \Sigma\bar{C}_H$$

where the Δ and Σ symbols have special meaning defined in the Notation section, and F and $\bar{C}_H$ are given by (8d) and (8e).

It may be noted from (8b) that the derivative of side force with rolling velocity will be similar to the derivative with sideslip, except by a factor and the addition of another term related to the $\dot{\phi}$ term in (2c). It may therefore be written by inspection that

9a) $$\frac{\partial\bar{C}_Y}{\partial\dot{\phi}} = \frac{h_0}{V}\frac{\partial\bar{C}_Y}{\partial\beta_s} - \Sigma\left[\frac{16}{\gamma\Omega}\left(\frac{\theta}{3}+\frac{3}{4}\lambda\right)\right]$$

It is apparent in (7b) that the derivative of side force with yawing velocity will be similar to the derivative with sideslip, except by a factor l_0/V, and a

change of signs for the rear rotor. By inspection of (8g), then

9b) $$\frac{\partial \bar{C}_Y}{\partial \dot{\psi}} = \mu \frac{l_o}{V} \left\{ \Sigma \left[\frac{F A_{1s}}{1+\frac{a\sigma}{8\mu}\left(1+\frac{\epsilon}{\epsilon_o}\right)} \right] - \frac{1}{2} \Delta \left[\frac{A_{1s}^2}{1+\frac{a\sigma}{8\mu}\left(1+\frac{\epsilon}{\epsilon_o}\right)} \right] \right\} - \frac{l_o}{V} \Delta \bar{C}_H$$

Rotor Rolling Moments

Rolling moments due to the rotor are of two kinds: those due to the side forces acting at h_o above the C.G. , and the hub moments associated with hinge offset. The former may be written immediately as

10a) $$\frac{\partial \bar{C}_l}{\partial \beta_s} = \frac{1}{2} \frac{h_o}{l_o} \frac{\partial \bar{C}_Y}{\partial \beta_s}$$

10b) $$\frac{\partial \bar{C}_l}{\partial \dot{\phi}} = \frac{1}{2} \frac{h_o}{l_o} \frac{\partial \bar{C}_Y}{\partial \dot{\phi}}$$

10c) $$\frac{\partial \bar{C}_l}{\partial \dot{\psi}} = \frac{1}{2} \frac{h_o}{l_o} \frac{\partial \bar{C}_Y}{\partial \dot{\psi}}$$

The hub moments can be written, by adaptation from Reference 3, as

11a) $$\bar{C}_l = \mp \frac{e}{l} \left\{ -\frac{\mu}{2}(\theta_o + \lambda) \mp \frac{1}{4}\mu \beta_o \left(\beta_s \pm \frac{l_o}{V}\dot{\psi} + \frac{h_o}{V}\dot{\phi} \right) + \frac{1}{6}(a_1)_i + \frac{(b_{1s})_i}{\gamma_2} \right\}$$

The derivatives of this, using equation (6) in conjunction with (7), etc. are

11b)

$$\frac{\partial \bar{C}_\ell}{\partial \beta_s} = -\frac{e}{\ell}\left\{\left(\frac{1}{6} - \frac{2}{9}\frac{\gamma}{\gamma_2}K\right)\mu^2 \Delta\left[\frac{A_{1s}}{1+\frac{a\sigma}{8\mu}\left(1+\frac{\epsilon}{\epsilon_0}\right)}\right] + \frac{1}{4}\mu \Sigma \beta_0 + \frac{1}{\gamma_2}\Sigma a_1 - \frac{1}{6}\Sigma b_1\right\}$$

11c)

$$\frac{\partial \bar{C}_\ell}{\partial \dot{\phi}} = \frac{h_0}{V}\frac{\partial \bar{C}_\ell}{\partial \beta_s} - \Sigma \frac{8}{\Omega \gamma \gamma_2}\left(\frac{e}{l_0}\right)$$

11d)

$$\frac{\partial \bar{C}_\ell}{\partial \dot{\psi}} = -\frac{l_0}{V}\frac{e}{\ell}\left\{\left(\frac{1}{6} - \frac{2}{9}\frac{\check{u}}{\gamma_2}K\right)\mu^2 \Sigma\left[\frac{A_{1s}}{1+\frac{a\sigma}{8\mu}\left(1+\frac{\epsilon}{\epsilon_0}\right)}\right] + \frac{1}{4}\mu \Delta \beta_0 + \frac{1}{\gamma_2}\Delta a_1 - \frac{1}{6}\Delta b_1\right\}$$

Equations of Motion

The linearized equations of motion for side force, rolling and yawing moments, can be written as follows:

15a) $$\left(\bar{C}_{Y\beta_s} - \frac{2\mu}{a\sigma} D\right)\beta_s + \left(\frac{\bar{C}_{Yr}}{\mu_*} - \frac{2\mu}{a\sigma}\right)D\psi$$

$$+\left(\Sigma\bar{C}_T + \left[\frac{2\mu}{a\sigma}\alpha_0 + \frac{\bar{C}_{Yp}}{\mu_*}\right]D\right)\phi = -\bar{C}_{Y\delta_a}\delta_a - \bar{C}_{Y\delta_r}\delta_r$$

15b) $$\bar{C}_{l\beta_s}\beta_s \quad \left(\frac{\bar{C}_{lr}}{\mu_*} + h_{xz} D\right)D\psi + \left(\frac{\bar{C}_{lp}}{\mu_*}D - h_x D^2\right)\phi =$$

$$= -\bar{C}_{l\delta_a}\delta_a - \bar{C}_{l\delta_r}\delta_r$$

15c) $$\bar{C}_{n\beta_s}\beta_s + \left(\frac{\bar{C}_{nr}}{\mu_*} - h_z D\right)D\psi + \left(\frac{\bar{C}_{np}}{\mu_*}D + h_{xz} D^2\right)\phi =$$

$$= -\bar{C}_{n\delta_a}\delta_a - \bar{C}_{n\delta_r}\delta_r$$

In these equations, the inertia coefficients are:

$$D(\ \ ) \equiv \frac{d(\ \ )}{d(t/\tau)} \quad ; \quad \tau = \frac{m}{\rho \pi R^2 \Omega R}$$

$$\mu_* \equiv \frac{m}{\rho \pi R^2 \ell_0} \quad ; \quad h_X \equiv \frac{1}{a\sigma} \frac{(k_X/\ell_0)^2}{\mu_*}$$

$$h_3 \equiv \frac{1}{a\sigma} \frac{(k_3/\ell_0)^2}{\mu_*} \quad ; \quad h_{X3} \equiv \frac{1}{a\sigma} \left(\frac{I_{X3}}{\mu_* m \ell_0^2} \right)$$

and α_0 is the initial angle of attack of the X axis.

Expressions for the aerodynamic derivatives in equations (15), from the previous work, are summarized below:

16a) $$\bar{C}_{Y\beta_s} = \mu \Delta \left[\frac{F A_{1s}}{1 + \frac{a\sigma}{8\mu}\left(1 + \frac{\epsilon}{\epsilon_0}\right)} \right] - \frac{1}{2}\mu \sum \left[\frac{A_{1s}^2}{1 + \frac{a\sigma}{8\mu}\left(1 + \frac{\epsilon}{\epsilon_0}\right)} \right]$$

$$- \sum \bar{C}_H + \mu^2 \left(\bar{C}_{Y\beta_s}\right)_{fus}$$

where F is given by (8d), $\bar{C}_H$ by (8e), and $(\bar{C}_Y)_{fus}$ by:

$$(\bar{C}_Y)_{fus} = \frac{2}{a\sigma} \left(\frac{Y_{fus}}{\rho V^2 \pi R^2} \right)$$

16b) $$\bar{C}_{Y_r} \equiv \frac{\partial \bar{C}_Y}{\partial \frac{l_o \dot{\psi}}{\Omega R}} =$$

$$= \Sigma \left[\frac{F A_{1S}}{1 + \frac{a\sigma}{8\mu}\left(1 + \frac{\epsilon}{\epsilon_o}\right)} \right] - \frac{1}{2} \Delta \left[\frac{A_{1S}^2}{1 + \frac{a\sigma}{8\mu}\left(1 + \frac{\epsilon}{\epsilon_o}\right)} \right]$$

$$- \frac{1}{\mu} \Delta \bar{C}_H$$

16c) $$\bar{C}_{Y_p} \equiv \frac{\partial \bar{C}_Y}{\partial \frac{l_o \dot{\phi}}{\Omega R}} = \frac{h_o}{l_o} \left\{ \Delta \left[\frac{F A_{1S}}{1 + \frac{a\sigma}{8\mu}\left(1 + \frac{\epsilon}{\epsilon_o}\right)} \right] \right.$$

$$\left. - \frac{1}{2} \Sigma \left[\frac{A_{1S}^2}{1 + \frac{a\sigma}{8\mu}\left(1 + \frac{\epsilon}{\epsilon_o}\right)} \right] - \frac{1}{\mu} \Sigma \bar{C}_H \right\}$$

$$- \frac{16R}{\gamma l_o} \Sigma \left(\frac{\theta_o}{3} + \frac{3}{4}\lambda \right)$$

16d) $$\bar{C}_{Y_{\delta a}} = K_a \Sigma \bar{C}_T$$

16e) $$\bar{C}_{Y_{\delta r}} = -K_r \Delta \bar{C}_T$$

17a) $$\bar{C}_{\ell\beta_s} = \frac{1}{2}\frac{h_o}{\ell_o}\left\{\mu\Delta\left[\frac{FA_{1s}}{1+\frac{a\sigma}{8\mu}\left(1+\frac{\epsilon}{\epsilon_o}\right)}\right] - \frac{1}{2}\mu\Sigma\left[\frac{A_{1s}^2}{1+\frac{a\sigma}{8\mu}\left(1+\frac{\epsilon}{\epsilon_o}\right)}\right] - \Sigma\bar{C}_H\right\}$$
$$- \frac{e}{2\ell_o}\left\{\left(\frac{1}{6} - \frac{2}{9}\frac{\gamma}{\gamma_2}K\right)\mu^2\Delta\left[\frac{A_{1s}}{1+\frac{a\sigma}{8\mu}\left(1+\frac{\epsilon}{\epsilon_o}\right)}\right] + \frac{1}{4}\mu\Sigma\beta_o + \frac{1}{\gamma_2}\Sigma a_1 - \frac{1}{6}\Sigma b_1\right\}$$
$$+ \mu^2\left(\bar{C}_{\ell\beta_s}\right)_{fus}$$

where

$$\left(\bar{C}_\ell\right)_{fus} \equiv \frac{2}{a\sigma}\frac{L_{fus}}{\rho V^2 \pi R^2 (2\ell_o)}$$

17b) $$\bar{C}_{l_r} \equiv \frac{\partial \bar{C}_l}{\partial \frac{\dot{\psi} l_o}{\Omega R}} = \frac{1}{2}\frac{h_o}{l_o}\left\{ \Sigma \left[\frac{F A_{1s}}{1+\frac{a\sigma}{8\mu}\left(1+\frac{\epsilon}{\epsilon_o}\right)}\right] - \frac{1}{2}\Delta\left[\frac{A_{1s}^2}{1+\frac{a\sigma}{8\mu}\left(1+\frac{\epsilon}{\epsilon_o}\right)}\right] - \frac{1}{\mu}\Delta \bar{C}_H \right\}$$

$$- \frac{1}{2}\frac{e}{l_o}\left\{ \mu\left(\frac{1}{6} - \frac{2}{9}\frac{\gamma}{\gamma_2}K\right)\Sigma\left[\frac{A_{1s}}{1+\frac{a\sigma}{8\mu}\left(1+\frac{\epsilon}{\epsilon_o}\right)}\right] + \frac{1}{4}\Delta\beta_o + \frac{1}{\gamma_2\mu}\Delta a_1 - \frac{1}{6\mu}\Delta b_1 \right\} + \mu^2 \left(\bar{C}_{l_r}\right)_{fus}$$

where $$\left(\bar{C}_{l_r}\right)_{fus} = \frac{\partial \bar{C}_{l_{fus}}}{\partial \frac{l_o \dot{\psi}}{\Omega R}} = -\frac{1}{2\sigma\mu}\left(\frac{h_t}{l_o}\right)\left(\frac{a_t}{a}\right)\left(\frac{S_t}{\pi R^2}\right)$$

and h_t is the distance, in feet, above the X axis of tail aerodynamic center.

17c) $$\bar{C}_{l_p} \equiv \frac{\partial \bar{C}_l}{\partial \frac{l_o \dot{\phi}}{\Omega R}} = \frac{1}{2}\left(\frac{h_o}{l_o}\right)^2 \left\{ \Delta\left[\frac{F A_{1s}}{1+\frac{a\sigma}{8\mu}\left(1+\frac{\epsilon}{\epsilon_o}\right)}\right] - \Sigma\left[\frac{A_{1s}^2}{1+\frac{a\sigma}{8\mu}\left(1+\frac{\epsilon}{\epsilon_o}\right)}\right] - \frac{1}{\mu}\Sigma \bar{C}_H \right\} - \Sigma \frac{R}{l_o}\frac{e}{l_o}\frac{8}{\gamma\gamma_2}$$

$$- \frac{1}{2}\frac{e}{l_o}\frac{h_o}{l_o}\left\{ \mu\left(\frac{1}{6} - \frac{2}{9}\frac{\gamma}{\gamma_2}K\right)\Delta\left[\frac{A_{1s}}{1+\frac{a\sigma}{8\mu}\left(1+\frac{\epsilon}{\epsilon_o}\right)}\right] + \frac{1}{4}\Sigma\beta_o + \frac{1}{\mu\gamma_2}\Sigma a_1 - \frac{1}{6\mu}\Sigma b_1 \right\}$$

$$- \frac{8 R h_o}{\gamma l_o^2}\Sigma\left(\frac{\theta_o}{3} + \frac{3}{4}\lambda\right)$$

17d) $$\bar{C}_{l_{\delta a}} = \frac{1}{2} K_a \frac{h_o}{l_o} \Sigma \bar{C}_T + \frac{1}{\gamma_2} \frac{e}{l_o}$$

17e) $$\bar{C}_{l_{\delta r}} = -\frac{1}{2} K_r \frac{h_o}{l_o} \Delta \bar{C}_T$$

18a) $$\bar{C}_{n_{\beta_s}} = \frac{\mu}{2} \Sigma \left[\frac{F A_{1S}}{1 + \frac{a\sigma}{8\mu}\left(1 + \frac{\epsilon}{\epsilon_o}\right)} \right] - \frac{\mu}{4} \Delta \left[\frac{A_{1S}^2}{1 + \frac{a\sigma}{8\mu}\left(1 + \frac{\epsilon}{\epsilon_o}\right)} \right] - \frac{1}{2} \Delta \bar{C}_H + \mu^2 \left(\bar{C}_{n_{\beta_s}}\right)_{fus}$$

where $$\left(\bar{C}_n\right)_{fus} \equiv \frac{2}{a\sigma} \frac{N_{fus}}{\rho V^2 \pi R^2 (2 l_o)}$$

18b) $$\bar{C}_{n_r} \equiv \frac{\partial \bar{C}_n}{\partial \frac{l_o \dot{\psi}}{\Omega R}} = \frac{1}{2} \Delta \left[\frac{F A_{1S}}{1 + \frac{a\sigma}{8\mu}\left(1 + \frac{\epsilon}{\epsilon_o}\right)} \right] - \frac{1}{4} \Sigma \left[\frac{A_{1S}^2}{1 + \frac{a\sigma}{8\mu}\left(1 + \frac{\epsilon}{\epsilon_o}\right)} \right] - \frac{1}{2\mu} \Sigma \bar{C}_H + \left(C_{n_r}\right)_{fus} \mu^2$$

where $$\left(\bar{C}_{n_r}\right)_{fus} = \frac{\partial \bar{C}_{n\,fus}}{\partial \frac{l_o \dot{\psi}}{\Omega R}} \doteq -\frac{1}{2\sigma} \left(\frac{a_t}{a}\right) \left(\frac{S_t}{\pi R^2}\right) \frac{1}{\mu}$$

18c) $$\bar{C}_{n_p} = \frac{\partial \bar{C}_n}{\partial \frac{l_o \dot{\phi}}{\Omega \zeta}} = \frac{1}{2} \frac{h_o}{l_o} \left\{ \Sigma \left[\frac{F A_{1s}}{1 + \frac{a\sigma}{8\mu}\left(1 + \frac{\epsilon}{\epsilon_o}\right)} \right] - \frac{1}{2} \Delta \left[\frac{A_{1s}^2}{1 + \frac{a\sigma}{8\mu}\left(1 + \frac{\epsilon}{\epsilon_o}\right)} \right] - \frac{1}{\mu} \Delta \bar{C}_H \right\} - \frac{8}{\gamma} \frac{R}{l_o} \Delta \left(\frac{\theta_o}{3} + \frac{3}{4}\lambda \right)$$

18d) $$\bar{C}_{n_{\delta a}} = \frac{1}{2} K_a \Delta \bar{C}_T$$

18e) $$\bar{C}_{n_{\delta r}} = -\frac{1}{2} K_r \Sigma \bar{C}_T$$

Although the foregoing equations look somewhat formidable, it will be seen that they involve many factors in common, so that their computation is not, in fact, overly lengthy. It is to be noted further, in the next section, that according to preliminary calculations at least, considerable additional simplification is permissible for typical configurations.

Sample Calculations

Preliminary calculations, for the HUP-1 helicopter without vertical fins, have indicated that several simplifications of the above equations may, in general, be permissible. These are tentatively listed as follows:

19a) $$\bar{C}_{Y_{\beta_s}} \doteq -\sum \bar{C}_H + \mu^2 \left(\bar{C}_{Y_{\beta_s}}\right)_{fus}$$

19b) $$\bar{C}_{Y_r} \doteq 0$$

19c) $$\bar{C}_{Y_p} \doteq 0$$

19d) $$\bar{C}_{l_{\beta_s}} \doteq -\frac{1}{2}\frac{h_o}{l_o}\sum \bar{C}_H - \frac{1}{2}\frac{e}{l_o}\left(\frac{1}{4}\mu \sum \beta_o + \frac{1}{\gamma_2}\sum a_1 - \frac{1}{6}\sum b_1\right)$$
$$+ \mu^2 \left(\bar{C}_{l_{\beta_s}}\right)_{fus}$$

19e) $$\bar{C}_{l_r} \doteq \mu^2 \left(\bar{C}_{l_r}\right)_{fus}$$

19f) $$\bar{C}_{l_p} \doteq -\frac{1}{2\mu}\left(\frac{h_o}{l_o}\right)^2 \sum \bar{C}_H - \frac{8}{\gamma\gamma_2}\left(\frac{R}{l_o}\right)\left(\frac{e}{l_o}\right)$$
$$- \frac{1}{2\mu}\left(\frac{e}{l_o}\right)\left(\frac{h_o}{l_o}\right)\left(\frac{1}{4}\mu \sum \beta_o + \frac{1}{\gamma_2}\sum a_1 - \frac{1}{6}\sum b_1\right)$$
$$- \frac{8 R h_o}{\gamma l_o^2}\sum\left(\frac{\theta_o}{3} + \frac{3}{4}\lambda\right)$$

19g) $\bar{C}_{n\beta_s} \doteq \mu^2 \; (\bar{C}_{n\beta_s})_{fus}$

19h) $\bar{C}_{n_r} \doteq -\frac{1}{2\mu} \Sigma \bar{C}_H + (\bar{C}_{n_r})_{fus} \, \mu^2$

19i) $\bar{C}_{n_p} \doteq 0$

where

19j) $\bar{C}_H \doteq \frac{\delta}{2a} \mu + \frac{8}{9} \mu \, \theta_o^2 + \frac{13}{6} \mu \theta_o \lambda + \frac{3}{2} \mu \lambda^2$

19k) $a_1 \doteq \frac{8}{3} \mu \, \theta_o + 2 \mu \lambda$

19m) $b_1 \doteq \frac{4}{3} K \mu \beta_o$

A few preliminary calculations of characteristic modes can be reported at this time. The helicopter considered was the HUP-1. Tentative results are as follows:

1) The lateral static stability is positive (+ dihedral effect), $\bar{C}_{\ell_\beta} < 0$

2) ~~The directional static stability is negative, $\bar{C}_{n\beta} < 0$, due to unstable fuselage contribution.~~

3) Damping in yaw and roll are positive ($\bar{C}_{n_r}$ and $\bar{C}_{\ell_p} < 0$).

4) The characteristic modes of motion for these derivatives consist of two stable real roots (simple convergences), and an unstable oscillation. Both real roots increase with increase in μ . The period of the oscillation increases slightly with μ , whereas the damping (cycles to double amplitude) is roughly independent of μ .

Studies of the coefficients of the characteristic equation indicate that for the normal HUP-1 configuration, the coefficients can be simplified to:

20a) $$C_4 \lambda^4 + C_3 \lambda^3 + C_2 \lambda^2 + C_1 \lambda + C_0 = 0$$

where

20b) $$C_4 \doteq 1$$

20c) $$C_3 \doteq - \frac{\bar{C}_{l_\beta}}{\hslash_x \mu_*}$$

20d) $$C_2 \doteq \frac{\bar{C}_{n_\beta}}{\hslash_3}$$

20e) $$C_1 \doteq - \frac{\bar{C}_{n_\beta}}{\hslash_3} \left(\frac{\bar{C}_{l_p}}{\hslash_x \mu_*} \right) - \frac{a\sigma}{2\mu} \bar{C}_T \frac{\bar{C}_{l_\beta}}{\hslash_x}$$

20f) $$C_0 \doteq \frac{a\sigma}{2\mu} \bar{C}_T \frac{\bar{C}_{l_\beta}}{\hslash_x} \left(\frac{\bar{C}_{n_r}}{\hslash_3 \mu_*} \right)$$

Solutions of this simplified characteristic equation gave approximately the same roots as the more complete treatment involving much more complex expressions.

It is evident that the helicopter can not be dynamically stable with $\bar{C}_{n_\beta} < 0$. If $\bar{C}_{n_\beta}$ is made positive (by addition of vertical tails, for example), all coefficients above in (20) will become positive, and the stability will be determined by Routh's Discriminant,

21a) $$R = C_3 C_2 C_1 - C_4 C_1^2 - C_3 C_0^2$$

which must exceed zero for stability. With approximate coefficients of (20) however,

21b) $$R = \frac{\sigma\sigma}{2\mu} \bar{C}_T \left(\frac{\bar{C}_{l\beta}}{h_x}\right) \left[- \frac{\bar{C}_{lp}}{h_x \mu_*} \frac{\bar{C}_{n\beta}}{h_3} - \frac{\sigma\sigma}{2\mu} \bar{C}_T \left(\frac{\bar{C}_{l\beta}}{h_x}\right) - \left(\frac{\bar{C}_{lp}}{h_x \mu_*}\right)^2 \left(\frac{\bar{C}_{nr}}{h_x \mu_*}\right)\right]$$

in which, for $\bar{C}_{n\beta} > 0$, all terms contribute negatively. This denies hope of stabilizing the lateral oscillation merely by providing static directional stability.

Solutions of the more complete equation have shown that, as $\bar{C}_{n\beta}$ is made more and more stable (positive), the divergence of the oscillation is reduced, but not overcome; and the period is reduced.

For large variations in the other derivatives, the approximate equation (20) is not valid. More complete solutions have shown, however, the following effects:

1) Reduction of $\bar{C}_{l\beta}$ has a favorable effect on the unstable oscillation, and particularly in combination with positive $\bar{C}_{n\beta}$, can possibly stabilize the lateral oscillation.

2) Increase of damping in yaw and roll, particularly the latter, is beneficial.

3) Increase of side force derivative $\bar{C}_{Y\beta}$ is beneficial, though possibly impractical.

Preliminary conclusions are that design changes to improve the lateral oscillatory instability should aim to a) provide static directional stability, b) reduce dihedral effect, c) improve damping in roll. Further calculations, in greater detail, of the practical possibilities are currently being undertaken, in addition to experimental verification of the static stability derivatives and dynamic response characteristics discussed above.

Conclusions

Lateral and directional static and dynamic stability derivatives are formulated for a tandem helicopter in forward flight. Equations of motion are written, from which the dynamic stability characteristics are predictable. These theoretical results are subject to experimental verification, and should be regarded as tentative.

Sample calculations for the HUP-1 helicopter indicate negative static directional stability, with an unstable oscillatory mode of lateral motion. It appears that provision of positive static stability alone can not stabilize the dynamic oscillatory mode; and that, in addition, reduced dihedral effect and increased damping in roll are required.

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